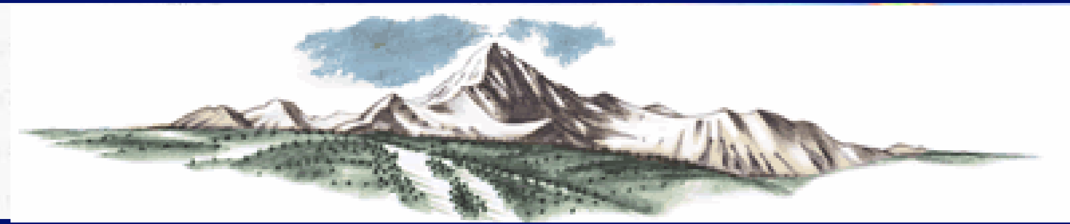

Truchas Flow Models

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Flow Model Requirements

- *Time-accurate solution of the Navier-Stokes equations*
- *Multiple Materials (with widely varying densities)*
- *Complex three dimensional geometry / multiple size scales*
- *Varied boundary and initial conditions*
- *Verification and Validation*
- *Transient interface tracking*
- *Normal and tangential surface tension forces*
- *Transport of enthalpy and species*
- *Turbulent diffusion of momentum, enthalpy, species*
- *Buoyant flow (temperature, species)*
- *Mushy zone flow*
- *Significant density changes*
- *Coupling to thermal state / stress distortion*

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Mesh Geometry and Variables

- ***Three dimensional (x, y, z) coordinates***
- ***Internal orthogonal mesh generator***
- ***Non-orthogonal mesh reader (multi formats)***
  - *Hexahedra, prisms, tetrahedra*
- ***Co-located, cell-centered variables***
  - *Face velocities for fluxing*
- ***Automatic Mesh Partitioning for 2 level preconditioning (Chaco)***
- ***"Monte Carlo" initialization of material bodies***

Flow solution within the overall time step advancement

- ***Flow starts each cycle***
  - *Preliminary step on first cycle (solenoidal velocity)*
- ***Flow may limit time step size***
- ***Enthalpy advection added at the start of the enthalpy solution***
- ***Solidification also changes material and species concentrations***

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Continuum Equations

$$\frac{\partial}{\partial t}\left(\rho\vec{u}\right)+\nabla\cdot\left(\rho\vec{u}\vec{u}\right)=\vec{F}_S+\vec{F}_D+\vec{F}_L+\rho'\vec{g}+\nabla\cdot\overleftrightarrow{\tau}$$

$$\nabla\cdot\vec{u}=0 \qquad \overleftrightarrow{\tau}=\mu_t\nabla\vec{u}-P\overleftrightarrow{I}$$

$$\frac{\partial\rho}{\partial t}+\vec{u}\cdot\nabla\rho=0$$

$$\vec{F}_D=-\overleftrightarrow{c}_d\cdot\vec{u} \qquad \overleftrightarrow{c}_d=\frac{(1-f^2)}{f^3}\overleftrightarrow{I}\cdot\vec{C}_p$$

Fractional-Step Solution Order

- ***Interface reconstruction and volume tracking***
  - *Advect momentum, enthalpy, species*
- ***Partially implicit velocity prediction***
- ***Transfer to face (fluxing) velocities***
  - *Rhie-Chow procedure*
- ***Cell–centered projection solution***
  - *Solenoidal fluxing velocity field*
- ***Correct cell-centered velocity vector***

Volume Tracking Formalism

$$\frac{\partial f_k}{\partial t} + \boldsymbol{u} \bullet \nabla f_k = 0 \quad \Rightarrow \quad \frac{\partial f_k}{\partial t} + \nabla \bullet \left(\boldsymbol{u} f_k \right) = f_k \nabla \bullet \boldsymbol{u}$$

$$V_k = \int f_k \, dV$$

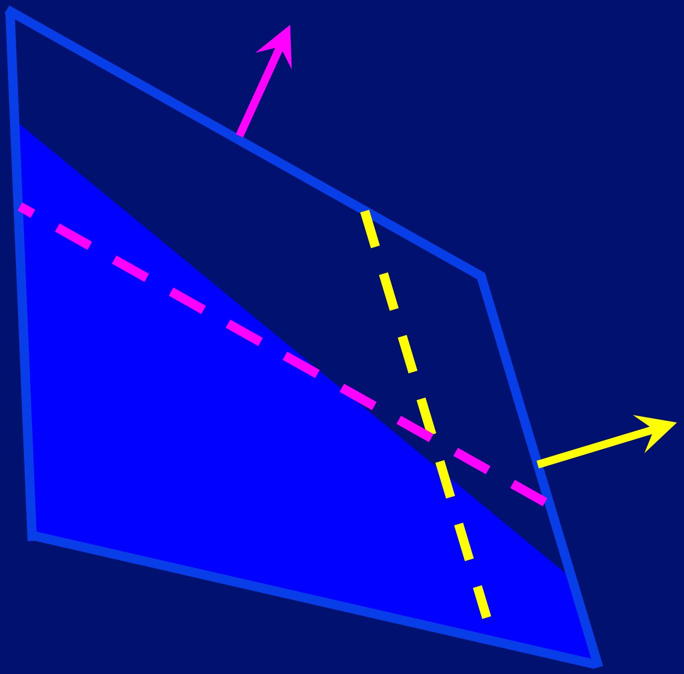
$$\frac{\partial V_k}{\partial t} + \int \nabla \bullet \left(\boldsymbol{u} f_k \right) dV = \int f_k \left(\nabla \bullet \boldsymbol{u} \right) dV$$

$$V_k^{n+1} - V_k^n + \sum_f \delta V_{k,f}^n = 0$$

Piecewise Linear Reconstruction

➢ *Construct planar interfaces within each cell*

- *Normal estimated by Green-Gauss*



- *sub cycling*

Predictor Step for the Cell-Centered Velocity

$$\rho_c^{n+1}\vec{u}_c^{\,*} = \rho_c^n \vec{u}_c^{\,n} + \delta t \left[\rho_c^{n+1}\vec{u}_f^{\,n} \bullet \nabla \vec{u}_c^{\,n} - \rho_c^{n+1} \left\langle \frac{\nabla P^n}{\rho_f^n} - \frac{\rho_f^n + \delta\rho_f^n}{\rho_f^n}\vec{g} \right\rangle_c + \nabla \cdot \overleftrightarrow{\tau}_t + \vec{F}_D \right]$$

$$\vec{F}_D = -\overleftrightarrow{c}_d \cdot \vec{u}^{\,*}$$

$$\nabla \cdot \overleftrightarrow{\tau}_t = \nabla \cdot \mu_t^{n+1} \nabla \left(\theta \vec{u}^{\,*} + (1-\theta)\vec{u}^{\,n} \right)$$

- ***Solve using linear solver as necessary***

Transfer cell-centered velocity to face centroids

- Use a Rhie-Chow like approach
  - Pressure gradient and Boussinesq force

$$\vec{u}_c^{\,*} + \delta t \left\langle \frac{\nabla P^n}{\rho_f^n} - \frac{\rho_f^n + \delta\rho_f^n}{\rho_f^n}\,\vec{g} \right\rangle_c \xrightarrow{\;transfer\;} \vec{u}_f^{\,\dagger}$$

$$\vec{u}_f^{\,*} = \vec{u}_f^{\,\dagger} - \delta t \left[\frac{\nabla P^n}{\rho_f^{n+1}} - \frac{\rho_f^n + \delta\rho_f^n}{\rho_f^{n+1}}\,\vec{g} \right]_f$$

Cell-centered projection step

- ***Project the face velocities onto a Solenoidal field***
- ***Solve for the <u>change</u> in pressure***

$$\vec{u}_f^{\,n+1} = \vec{u}_f^{\,*} - \delta t \left[\frac{\nabla \delta P^{n+1}}{\rho_f^{n+1}} - \left(\frac{\rho_f^{n+1} + \delta\rho_f^{n+1}}{\rho_f^{n+1}} - \frac{\rho_f^{n} + \delta\rho_f^{n}}{\rho_f^{n+1}} \right) \vec{g} \right]_f$$

$$= \vec{u}_f^{\,*} + \delta t \left(\frac{\rho_f^{n+1} + \delta\rho_f^{n+1}}{\rho_f^{n+1}} - \frac{\rho_f^{n} + \delta\rho_f^{n}}{\rho_f^{n+1}} \right)_f \vec{g} - \delta t \left(\frac{\nabla \delta P^{n+1}}{\rho_f^{n+1}} \right)_f$$

Cell-centered velocity correction

- *Update for the modified pressure and Boussinesq force*
- *Average these terms over all cell faces*

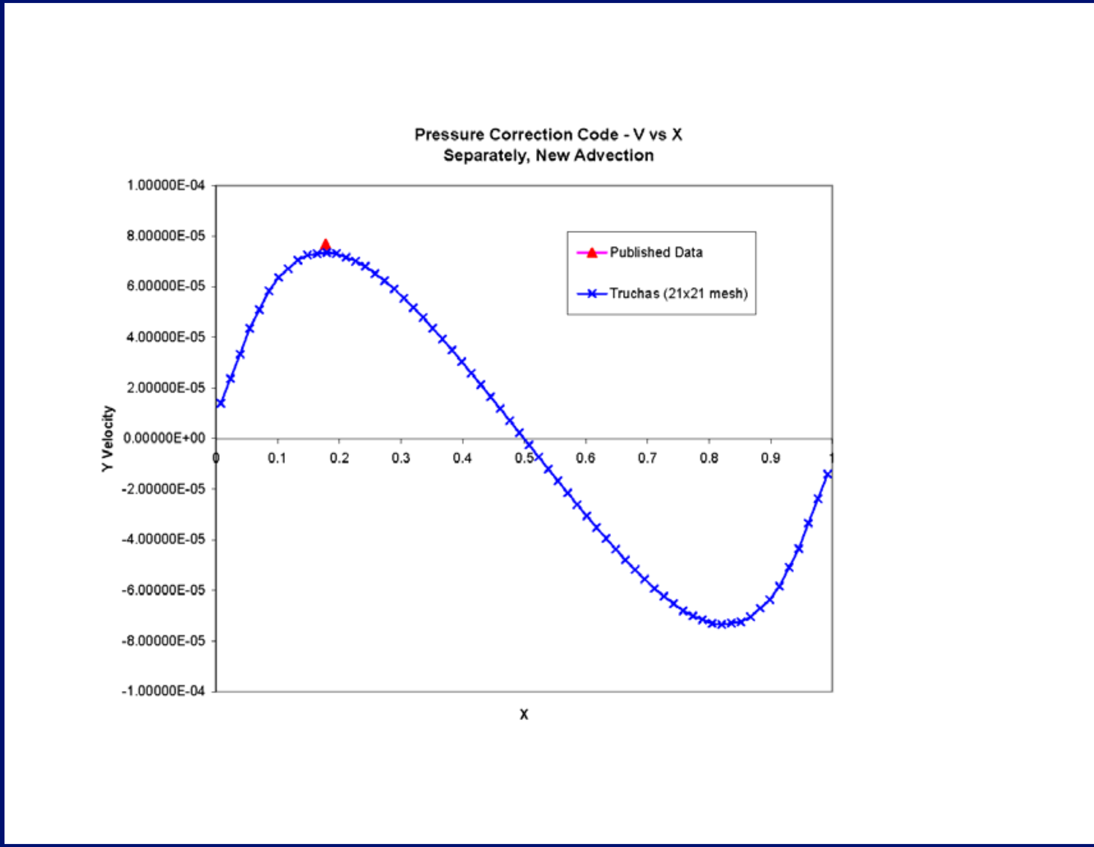
$$\frac{\vec{u}_c^{\,n+1} - \vec{u}_c^{\,*}}{\delta t} = -\left\langle \frac{\nabla \delta P^{n+1}}{\rho_f^{n+1}} - \left(\frac{\rho_f^{n+1} + \delta\rho_f^{n+1}}{\rho_f^{n+1}} - \frac{\rho_f^{n} + \delta\rho_f^{n}}{\rho_f^{n+1}} \right) \vec{g} \right\rangle_c$$

Verification / Validation

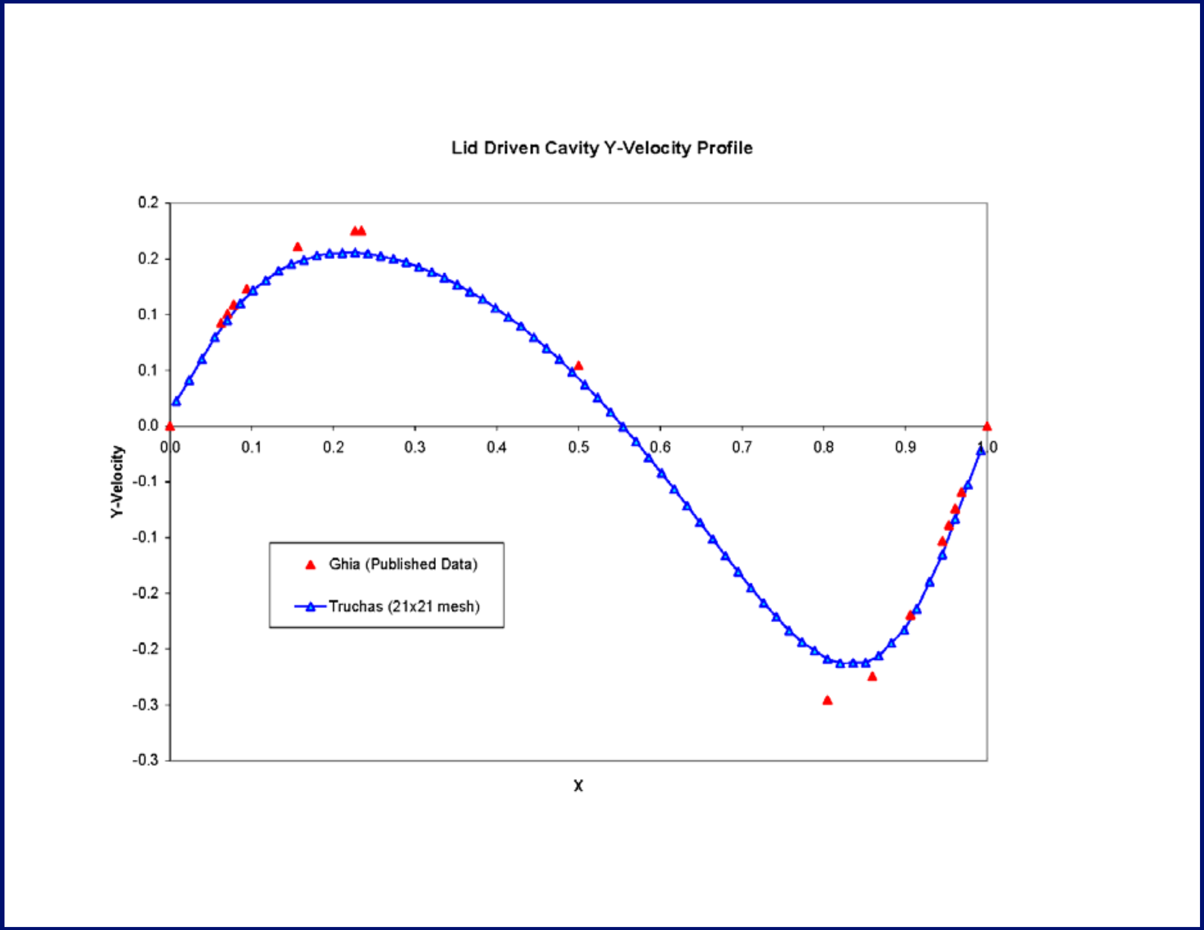
- *Poiseuille flow in ducts and pipes*
- *Backward facing step (low Re)*
- *de Vahl Davis natural convection benchmark*
- *Lid driven cavity benchmark (Ghia)*
- *Voller-Prakash solidifying flow*
- *Porous media pressure drop*
- *Baneczek freezing test*
- *SCN solidification Bencharm*

(Thanks to Jeff Marchetta for many of these calculations.)

de Vahl Davis benchmark vertical velocity comparison



Lid driven cavity horizontal velocity comparison



Future Directions

- *Modeling flow in the mushy zone*
- *Higher order spatial operators*
- *Local Runge-Kutta*
- *Reduced computing cost*
- *Turbulence model improvements*
- *Immersed boundary method*
- *Improved interface tracking*

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